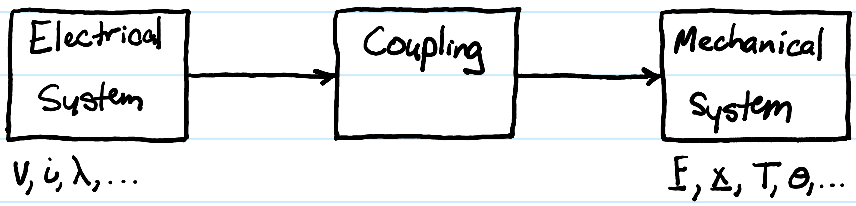
2018-10-15-1

Chapter 4

Structure of Systems:



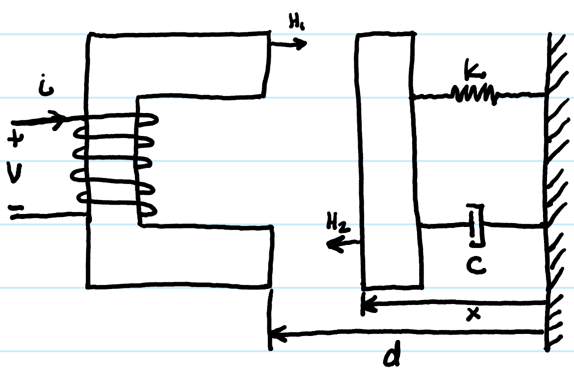
Governing Equations

Electrical System: Kirchhoff's laws

Mechanical System: Newton's 2^{nd} Law

Examine Electrical System First

Ex Translational System



Ampere's law: $H_1(d-x)+H_2(d-x)=Ni$

Gauss's law: $\mu_0 H_1 A-\mu_0 H_2 A=0 \Rightarrow H_1=H_2=H$

$$2H(d-x)=Ni \Rightarrow 2\frac{B}{\mu_0}(d-x)=Ni$$

$$\Rightarrow BA\left(\frac{2(d-x)}{\mu_0 A}\right)=Ni$$

$$\Rightarrow \phi\left(\frac{2(d-x)}{\mu_0 A}\right)=Ni$$

$$\boxed{\phi=\left[\frac{\mu_0 A N}{2(d-x)}\right]i}$$

$$\lambda=N\phi \Rightarrow \lambda=\left[\frac{\mu_0 A N^2}{2(d-x)}\right]i \Rightarrow \boxed{\lambda=L(x)i}$$

2018-10-15-2

$\lambda = L(x)i$

$V = \frac{d\lambda}{dt} \Rightarrow V = \frac{d}{dt}[L(x)i] \Rightarrow \boxed{V = L(x)\frac{di}{dt} + \frac{\partial L}{\partial x}\frac{dx}{dt}i}$

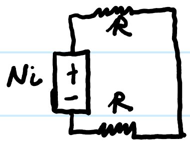
$L(x)\frac{di}{dt} \Rightarrow$ transformer voltage

$\frac{\partial L}{\partial x}\frac{dx}{dt}i \Rightarrow$ speed voltage

<u>Here:</u> $V = \frac{\mu_0 A N^2}{2(d-x)}\frac{di}{dt} + \frac{\mu_0 A N^2}{2(d-x)^2}\frac{dx}{dt}i$

* Note: if x is fixed, $\frac{dx}{dt} = 0$ and $L(x) = L$, giving $V = L\frac{di}{dt}$, which is the equation for an ideal inductor.

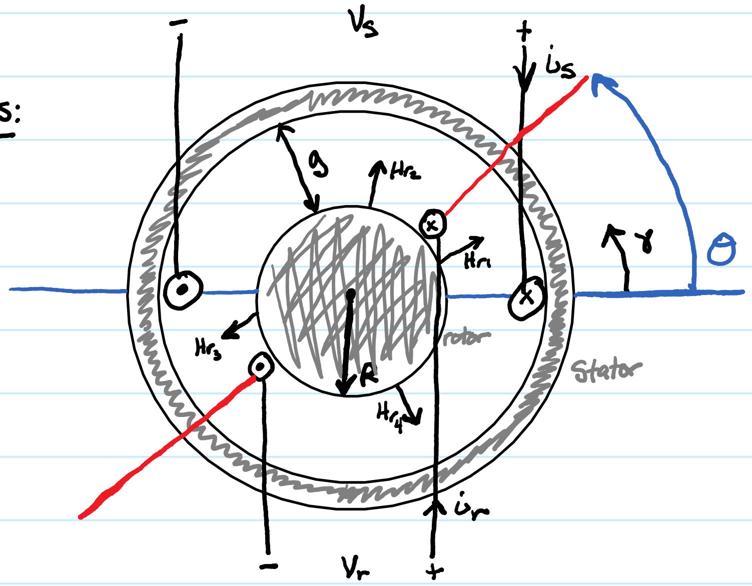
* Could also obtain from magnetic equivalent circuit



Rotational Systems:

Assume: $g \ll R$

depth: ℓ



$0<\gamma<\theta$	$\theta<\gamma<\pi$	$\pi<\gamma<\theta+\pi$	$\theta+\pi<\gamma<2\pi$
H_{r_1}	H_{r_2}	H_{r_3}	H_{r_4}

ACL:

$$H_{r_1}g - H_{r_4}g = N_s i_s \qquad H_{r_3}g - H_{r_2}g = -N_s i_s$$

$$H_{r_2}g - H_{r_1}g = N_r i_r \qquad H_{r_4}g - H_{r_3}g = -N_r i_r$$

GL: Enclose rotor

$$\mu_0 H_{r_1}(R\theta)\ell + \mu_0 H_{r_2} R(\pi-\theta)\ell + \mu_0 H_{r_3}(R\theta)\ell + \mu_0 H_{r_4} R(\pi-\theta)\ell = 0$$

$$\left[H_{r_1} + H_{r_3} - (H_{r_2}+H_{r_4})\right]\theta + (H_{r_2}+H_{r_4})\pi = 0$$

$$H_{r_2} = -H_{r_4}$$

$$H_{r_1} = -H_{r_3}$$

$$H_{r_1} - H_{r_4} = \frac{N_s i_s}{g} \Rightarrow H_{r_1} = H_{r_4} + \frac{N_s i_s}{g} \Rightarrow H_{r_1} = -H_{r_2} + \frac{N_s i_s}{g}$$

$$H_{r_2} - H_{r_1} = \frac{N_r i_r}{g}$$

$$H_{r_2} - \left(-H_{r_2} + \frac{N_s i_s}{g}\right) = \frac{N_r i_r}{g} \Rightarrow 2H_{r_2} = \frac{N_s i_s + N_r i_r}{g}$$

$$\boxed{\begin{aligned} H_{r_2} &= \frac{N_s i_s + N_r i_r}{2g} = -H_{r_4} \\ H_{r_1} &= \frac{N_s i_s - N_r i_r}{2g} = -H_{r_3} \end{aligned}}$$